Name: \_\_\_\_\_\_\_\_**SOLUTIONS**\_\_\_\_\_\_

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Full Test (Sections 1 and 2)

Total Time: 57 minutes

Total Marks: 50 marks

Student Result \_\_\_\_\_\_\_\_/ 50

**MATHEMATICS METHODS Unit 3/4**

**TEST 3 -2021: Logarithmic Functions, Continuous Random Variables**

**Calculator Free Section**

Time: 27 minutes

Total Marks: \_\_\_\_\_\_ / 24 marks

Resources allowed: SCSA Formula Sheet

**Instructions to candidates**

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks**. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.** If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

|  |  |
| --- | --- |
| **Question 1** | **[2, 2 = 4 marks]** |

Evaluate each of the following.

a)

**✓**

**✓**

b)

**✓**

**✓**

|  |  |
| --- | --- |
| **Question 2** | **[2 marks]** |

Express as a single logarithm.

**✓**

**✓**

|  |  |
| --- | --- |
| **Question 3** | **[2 marks]** |

Solve , for .

**✓** takes natural log of both sides

**✓** correct

|  |  |
| --- | --- |
| **Question 4** | **[2, 2 = 4 marks]** |

Find for each of the following.

a)

**✓** uses product rule

**✓** correct derivative

b)

**✓** uses log law

**✓** correct derivative

|  |  |
| --- | --- |
| **Question 5** | **[4 marks]** |

Below is the graph of .

Determine the shaded area trapped between the positive axes and .

<EFOFEX>
id:fxd{b86b0175-3939-4dfe-b71e-d4e9a6ddb36b}

FXData:
</EFOFEX>

Solve

**✓** solves for root

Area

**✓** indicates correct integral for area

**✓** integrates correctly

units2 **✓** correct area

|  |  |
| --- | --- |
| **Question 6** | **[2, 1, 3, 2 = 8 marks]** |

A continuous random variable, , has a probability density function given by that below.

a) Show that the value for the constant .

**✓** correct integral

**✓** correct integration

b) Determine

**✓** correct

c) Determine

**✓** correct integral

**✓** correct integration

**✓** correct

d) Determine the expected value, , for this continuous random variable.

**✓** correct integral

**✓** correct

End of Calculator Free Section

Name: \_\_\_\_\_\_\_\_**SOLUTIONS**\_\_\_\_\_\_

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**MATHEMATICS METHODS Unit 3/4**

**TEST 3 -2021: Logarithmic Functions, Continuous Random Variables**

**Calculator Assumed Section**

Time: 30 minutes

Total Marks: \_\_\_\_\_\_ / 26 marks

Resources allowed:

SCSA Formula Sheet

Up to three Calculators and

One A4 sheet, both sides of notes

**Instructions to candidates**

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. **For any question or part question worth more than two marks, valid working or justification is required to receive full marks.** If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

|  |  |
| --- | --- |
| **Question 7** | **[2, 2, 2 = 6 marks]** |

The probability density function that models the likely length of time, in minutes, between meteors (shooting stars) being seen during a meteor shower is given below.

a) Show that the probability of two meteors being seen within 1 minute of each other is approximately 0.5507.

**✓** correct integral

**✓** integrates correctly

b) Determine the probability that it takes more than 5 minutes between two consecutive meteors being seen.

**✓** correct integral, or

**✓** correct

c) Determine the expected time, or long term average, between meteors during the meteor shower.

**✓** correct integral

Using CAS

Expected average time between meteors is 1.25 minutes. **✓** correct

|  |  |
| --- | --- |
| **Question 8** | **[2, 3 = 5 marks]** |

The velocity, ms-1, of a particle P moving in a straight line, at time seconds, from a fixed origin O, is given by:

, for .

1. Comment on the nature of the velocity of the particle for .

The particle has a positive velocity for all , however, decreasing with time.

**✓** positive **✓** decreasing

1. If the particle initially has a displacement of metres, show that the displacement at any time , from the fixed origin O, is given by:

**✓** integrates velocity

When

**✓** determines const. of integration

**✓** uses log laws to show that which is required

|  |  |
| --- | --- |
| **Question 9** | **[2, 2, 3 = 7 marks]** |

The ear has the remarkable ability to handle an enormous range of sound levels. In order to express levels of sound meaningfully in numbers that are more manageable, a logarithmic scale is used, rather than a linear scale. This scale is the decibel (dB) scale.

The sound intensity level, , is given by the formula below:

dB where is the sound intensity and is the reference sound intensity.

and are measured in watt/m2.

1. Listening to a sound intensity of 1.26 billion times that of the reference intensity for more than 2 hours is considered unsafe. To what sound intensity level, , does this correspond?

**✓** substitutes for

dB **✓** calculates level

1. The reference sound intensity, , has a sound intensity level of 0 dB. If a petrol chainsaw has a sound intensity, watt/m2 and this corresponds to a sound intensity level of 110 dB, determine in watt/m2.

**✓** substitutes for and

watt/m2 **✓** correct

The distance from a sound source, , will affect sound intensity, , according to the following relationship.

A jet taking off at 300 metres away typically reaches 100 dB.

c) Calculate the sound intensity level, in dB, a jet take-off will typically reach at 1 km away.

**✓** expresses the relationship between the two sound intensities based on distance from sound source

**✓** determines sound intensity at 1 km (in terms of )

dB **✓** determines sound intensity level at 1 km

|  |  |
| --- | --- |
| **Question 10** | **[2, 3, 1, 2 = 8 marks]** |

A continuous random variable , has the following probability density function.

a) Show that the value of .

**✓** correct integral

**✓** correct evaluation of definite integral

b) Determine the cumulative density function for the continuous random variable .

for **✓** correct integral

**✓** correct for ,

**✓** correct for and

The mean or expected value for the continuous random variable is .

c) Determine the value of , correct to 4 decimal places.

CAS **✓** correct

A second continuous random variable , is defined as .

d) Determine, correct to 2 decimal places, the value of:

i).

**✓** correct

ii).

**✓** correct

End of Test